RESEARCH SUMMARY

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1. Introduction

My general area of research is algebraic topology. Broadly speaking, this means that I am interested in connections between abstract algebra and topological spaces. Modern algebraic topology begins with the attempt to classify all possible topological spaces by algorithmically constructing certain abstract groups from each topological space. Manipulations of and relations between spaces translate into manipulations of and relations between abstract groups. To capture different structural aspects, multiple constructions moving between topological spaces and abstract algebraic objects have been identified. Questions about structure or existence of topological spaces are translated via these methods to questions about structure or existence of algebraic objects. We may also run reverse constructions – building topological spaces from abstract algebraic objects. This allows us to convert algebraic questions into topological questions, sometimes changing complicated algebraic problems into topological ones which are more tractable. Very basic examples of this are the proofs that subgroups of free groups are free, as well as the fundamental theorem of algebra.

My current research is divided into three projects. My primary project is continuing joint work with Dev Sinha at the University of Oregon on Lie coalgebras and related structures in algebraic topology. Related to this work, my second project is continuing work giving a new formulation of cooperads and coalgebras in general symmetric monoidal categories – with particular applications in the categories of ungraded ring modules and topological spaces. My third project is joint with Brian Munson at Wellesley College constructing a "calculus of graph functors" which we hope to use to streamline computations of chromatic numbers of graphs à la Babson and Kozlov. More long-term, I am interested in investigating further a new, deep connection between topological spaces and algebraic objects via spectra hinted at by recent work in Goodwillie's calculus of functors and topological operads. The three projects above each feed into this long term goal in a different manner.

Following is a very brief overview of these projects, with in-depth discussions in later sections.

1.1. **Lie coalgebras and related structures in algebraic topology.**

My joint work with Dev Sinha on Lie coalgebras and applications in topology began in late 2005. Our foundational work faced many obstacles which have only recently been overcome. Now that the foundations are complete, a broad vista of applications has opened.

In our first paper "Lie coalgebras and rational homotopy theory, I: graph coalgebras" we define a new category of coalgebras which we call graph coalgebras, we introduce a new approach to Lie coalgebras as quotients of graph coalgebras, and we apply this to classical rational homotopy theory. We show that the Harrison homology functor from commutative algebras to Lie coalgebras naturally factors through our category of graph coalgebras, and we use this factorization to explicitly display the duality between Harrison homology and homotopy via a pairing of graphs and trees. Our work leads to a new view of rational homotopy theory as well as a more computationally-friendly framework for Harrison homology and Lie coalgebras.

In our second paper "Lie coalgebras and rational homotopy theory, II: Hopf invariants" we revisit the geometry behind the Harrison/homotopy pairing of the first paper in order to give a new, geometric answer to the classical question "how can cochain data determine homotopy groups?" Explicitly we show that graphs decorated by cochain elements give a complete set of homotopy functionals compatible with Whitehead products, and we give a simple, geometric method for making calculations. Our work unifies and generalizes constructions given by Boardman-Steer, Sullivan, Haefliger, Hain and Novikov, using our graph coalgebraic viewpoint to incorporate both formalism and geometry.

Our work on graph coalgebras gives a radically different and highly computable view of an extremely fundamental object, Lie coalgebras. Initial work indicates a number of fruitful applications and extensions of our ideas ranging from rational homotopy theory with infinite fundamental group, to the word problem in group theory, to a generalized Hopf invariant one question.

1.2. **Cooperads and Coalgebras.** Considering graph coalgebras and the graph cooperad has led me to work on the foundations of cooperad theory. Operads are objects which encode algebra structures. For example there operads $\mathcal{L}ie$, $\mathcal{C}om$, $\mathcal{A}s$ encoding Lie, commutative, and associative algebra structure, as well as A_∞ and E_∞ operads encoding homotopy associative and homotopy commutative algebra structure. Recent work generalizes these notions to the topological setting and beyond. Dually, cooperads encode coalgebra structures. That is, operads tell how things are multiplied, cooperads tell how they are factored.

Operad theory is aided by the fact that in all categories of interest, operadic composition is associative. However, the dual notion of cooperadic cocomposition is associative in almost none of the standard categories of interest. One striking effect of this is the difficulty of describing general cofree coalgebras, a question which has garnered interest in the theoretical computer science community (see e.g. work of Berstel and Reutenauer and also Block and Griffin). In my paper "Cofree coalgebras over cooperads" I lay foundations for a new approach to cooperad theory using an associative universal composition product whose left and right Kan extensions give the operadic and cooperadic composition products. This is used to give a new construction of cofree coalgebras over cooperads improving the existing constructions of Smith, Fox, Hazewinkel, and Block-Griffin. Furthermore I give examples showing that my new foundations give a relatively easy way to give explicit descriptions of cooperad structures and maps between cooperads and make direct computations.

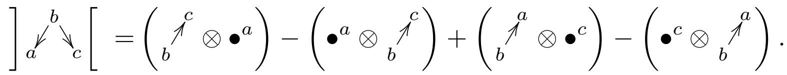
In further work, I plan to extend these constructions to give a new description of the cofree cooperad functor, valid in the categories of ungraded ring modules and also topological spaces. This would yield a new construction of the cooperadic cobar construction which preliminary calculations indicate would extend that of Michael Ching in the topological setting. There are a number of further projects, in my joint work with Sinha described above, and also investigating Goodwillie's homotopy calculus of functors, where such a construction could be useful.

1.3. **Calculus of graph functors.** My dissertation work was on Goodwillie's homotopy calculus of functors. In my dissertation I constructed a new homotopy calculus of functors in the rational homotopy categories of differential graded commutative coalgebras and differential graded Lie algebras. In a similar, joint project with Brian Munson at Wellesley College we are constructing a new calculus of graph functors (functors from the category of graphs to the category of topological spaces). Our goal is to use this calculus of graph functors to streamline and better understand the method of Lovász generalized by Babson and Kozlov in 2007 for computing bounds for chromatic numbers of graphs using algebraic topology. We have currently solved about half of the problems required to complete our calculus of graph functors construction, and expect to be able to complete our work and have it ready for publication within the next year. We expect our work to be of great interest both to homotopy theorists wanting to better understand Weiss' and Goodwillie's calculi of functors and to applied mathematicians and computer scientists interested in chromatic numbers of graphs.

2. Lie coalgebras and related structures in algebraic topology

2.1. **A brief introduction to Lie coalgebras and generalized Hopf invariants.**

Definition 2.1. Let V be a vector space. Define $\mathbb{G}(V)$ to be the span of the set of oriented acyclic graphs with vertices labeled by elements of V modulo multilinearity in the vertices. $\mathbb{G}(V)$ has an anti-commutative coproduct given by $]G[\, = \sum_e (G_1^{\hat{e}} \otimes G_2^{\hat{e}} - G_2^{\hat{e}} \otimes G_1^{\hat{e}})$, where e ranges over the edges of G, and $G_1^{\hat{e}}$ and $G_2^{\hat{e}}$ are the connected components of the graph obtained by removing e, which points to $G_2^{\hat{e}}$. For example,



We call $\mathbb{G}(V)$ the cofree graph coalgebra on V. The number of vertices in a graph is called its weight. Graph coalgebras pair with non-associative binary algebras via the configuration pairing between oriented acyclic graphs and binary trees described below.

Definition 2.2. Given G and T, an oriented acyclic graph with vertices $V = \{1, \ldots, n\}$ and a binary tree embedded in the upper half-plane with leaves $L = \{1, \ldots, n\}$, define

$$\beta_{G,T} : \{\text{edges of } G\} \longrightarrow \{\text{internal vertices of } T\}$$

by sending an edge from vertex i to j in G to the vertex at the nadir of the shortest path in T between the leaves i and j. Use $\beta_{G,T}$ to define the configuration pairing of G and T by

$$\langle G, T \rangle = \begin{cases} \prod_{\substack{e \text{ an edge} \\ \text{of } G}} \operatorname{sgn}\big(\beta_{G,T}(e)\big) & \text{if } \beta \text{ is surjective,} \\ 0 & \text{otherwise} \end{cases}$$

where $\operatorname{sgn}\big(\beta_{G,T}(e)\big) = \pm 1$ depending on whether the direction of e agrees with the ordering of the leaves of T induced by its planar embedding.

Example 1. Following is the map $\beta_{G,T}$ for two different binary trees T.

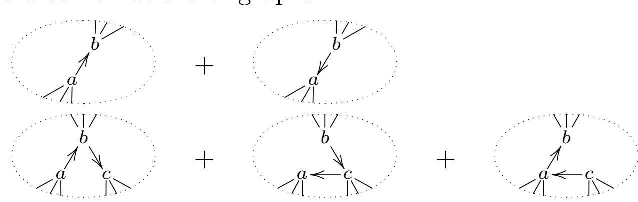
In the first example, $\operatorname{sgn}\big(\beta(e_1)\big) = -1$ and $\operatorname{sgn}\big(\beta(e_2)\big) = 1$. In the second example, $\operatorname{sgn}\big(\beta(e_1)\big) = 1$ and $\operatorname{sgn}\big(\beta(e_2)\big) = -1$.

The configuration pairing is so named because it naturally arises in the cohomology-homology pairing for configuration spaces. The graph coproduct and binary algebra product are dual in the configuration pairing. Further, the kernel of the configuration pairing is precisely the Jacobi and anti-symmetry subspace on the binary algebra side, and the "Arnold and arrow-reversing" subspace on the graph coalgebra side.

Definition 2.3. Define $\mathbb{E}(V)$ to be the quotient of $\mathbb{G}(V)$ by $\operatorname{Arn}(V)$, where $\operatorname{Arn}(V)$ is the subspace generated by arrow-reversing and Arnold combinations of graphs:

(arrow-reversing)

(Arnold)



Above a, b, and c are vertices of a graph which is fixed outside of the indicated area.

Theorem 2.4 (Sinha and W—)**.** *If V is graded in positive degrees, then $\mathbb{E}(V)$ is isomorphic to the cofree Lie coalgebra on V, with the graph coproduct on $\mathbb{G}(V)$ descending to the Lie cobracket on $\mathbb{E}(V)$.*

Indeed, if V and W are linearly dual, then the configuration pairing gives a perfect pairing between $\mathbb{E}(V)$ and the free Lie algebra on W with graph coproduct dual to Lie bracket.

Definition 2.5. Let (A, d_A, μ_A) be a one-connected differential graded algebra. The graph coalgebraic bar construction $\mathcal{G}(A)$ is the total complex of $\big(\mathbb{G}(s^{-1}\bar{A}),\ \tilde{d}_A,\ d_\mu\big)$. Here $s^{-1}\bar{A}$ is the desuspension of the ideal of positive-degree elements of A, $\tilde{d}_A$ is the extension of d_A by Leibniz, and $d_\mu(g)$ is a sum contracting each edge of g in turn, multiplying endpoints.

If A is commutative, the Lie coalgebraic bar construction $\mathcal{E}(A)$ is the quotient $\mathcal{G}(A)\big/\mathrm{Arn}(s^{-1}\bar{A})$.

Theorem 2.6 (Sinha and W—)**.** *$\mathcal{E}(A)$ is well defined and gives a model for the Harrison complex, or equivalently the André-Quillen homology of a commutative algebra. Thus, these constructions factor through the category of graph coalgebras.*

The origin of these graph complexes was the study of generalized Hopf invariants. Let X be a simplicial set, $A(X)$ be the PL-forms on X, $\mathcal{E}(X) = \mathcal{E}(A(X))$, and $H^n_{\mathcal{E}}(X) = H_n(\mathcal{E}(A(X)))$. Define $\mathcal{G}(X)$ and $H^n_{\mathcal{G}}(X)$ similarly.

Lemma 2.7. *$H^{n-1}_{\mathcal{E}}(S^n)$ is rank one, generated by a weight one cocycle.*

Definition 2.8. Given a cocycle $\gamma \in \mathcal{E}^{n-1}S^n$ we let $\tau(\gamma) \simeq \gamma$ be any cohomologous cocycle of weight one. Call $\tau(\gamma)$ a Hopf cocycle associated to γ.

Write $\int_{\mathcal{E}(S^n)}$ for the map from cocycles in $\mathcal{E}^{n-1}(S^n)$ to $\mathbb{Q}$ given by $\int_{\mathcal{E}(S^n)} \gamma = \int_{S^n} \tau(\gamma)$.

Lemma 2.9. *The map $\int_{\mathcal{E}(S^n)}$ is well defined and induces the isomorphism $H^{n-1}_{\mathcal{E}}(S^n) \cong \mathbb{Q}$.*

Definition 2.10. Given a cocycle $\gamma \in \mathcal{E}^{n-1}S^n$ and $f : S^n \to X$, the Hopf invariant $\eta_\gamma(f)$ of f with respect to γ is $\int_{\mathcal{E}(S^n)} f^*(\gamma)$.

Just as $\mathcal{E}$ factors through graph coalgebras, so too does $\int_{\mathcal{E}(S^n)}$ factor through $\int_{\mathcal{G}(S^n)}$ (which is defined in the same way as 2.8). In practice we compute Hopf invariants by picking graph representatives and calculating $\eta^{\mathcal{G}}_\gamma(f) = \int_{\mathcal{G}(S^n)} f^*\gamma$.

Example 2. Let ω be a generating 2-cocycle on S^2 and $f : S^3 \to S^2$. The graph $\gamma = -\underset{\omega}{\overset{\omega}{\nearrow}}$ is a cocycle in $\mathcal{G}(S^2)$ with $f^*\gamma = -\underset{f^*\omega}{\overset{f^*\omega}{\nearrow}}$. Because $f^*\omega$ is closed and of degree two on S^3, it is exact. Let $d^{-1}f^*\omega$ be a choice of a cobounding cochain. Then

$$d\left(\underset{d^{-1}f^*\omega}{\overset{f^*\omega}{\nearrow}}\right) = \underset{f^*\omega}{\overset{f^*\omega}{\nearrow}} \;+\; \big(d^{-1}f^*\omega \wedge f^*\omega\big).$$

Thus $f^*\gamma$ is homologous to $d^{-1}f^*\omega \wedge f^*\omega$. The corresponding Hopf invariant is $\int_{S^3} d^{-1}f^*\omega \wedge f^*\omega$, which is the classical formula for Hopf invariant given by Whitehead in 1947.

The formulae get more complicated in higher weights, but they are still computable and have the same basic ingredients – pulling back cochains, taking d^{-1} and products. In particular, if γ is defined using Thom forms of disjoint submanifolds W_i of a manifold X, then $\eta_\gamma(f)$ is a generalized linking number of the $f^{-1}(W_i)$.

Theorem 2.11 (Sinha and W—)**.** *Let X be simply connected. Then the map $H^n_{\mathcal{E}}(X) \to \mathrm{Hom}\big(\pi_{n+1}(X), \mathbb{Q}\big)$, which sends γ to the Hopf invariant associated to γ, is an isomorphism of Lie coalgebras. Furthermore the map above is descended from a coalgebra map $H^n_{\mathcal{G}}(X) \to \mathrm{Hom}\big(\pi_{n+1}(X), \mathbb{Q}\big)$.*

2.2. **Future work.** First, we plan to extend our approach to rational homotopy theory beyond the simply connected setting in characteristic zero. In preliminary calculations, it seems that our techniques will apply beyond the nilpotent setting, possibly even to infinite fundamental groups (extending work of Gómez-Tato, Halperin, and Thomas). Further, applying our techniques to $K(\pi,1)$'s could yield new insight into the lower central series and a new algorithm to attack the word problem for residually nilpotent groups. Hopf invariants in this case can be understood as a sort of "generalized linking number" of elements.

A second area of inquiry is the deeper setting of characteristic p. The main starting point for our Lie coalgebraic approach to homotopy theory was studying explicit Hopf invariants, and realizing that their values on Whitehead products were governed by the same pairing as governs the homology and cohomology of ordered configuration spaces, a phenomenon explained by iterated loopspace theory. At p, loopspace theory leads to studying the pairing between the cohomology and homology of symmetric groups.

Another closely related problem is our generalization of the Hopf invariant one question. Hopf invariants over the integers are defined for any space and give a full rank subgroup of $\mathrm{Hom}\big(\pi_n(X),\,\mathbb{Z}\big)$. Calculating the cokernel of the inclusion of this subgroup generalizes the classical Hopf invariant one question and could help us connect our approach in characteristic zero to topology in characteristic p.

3. Cooperads and coalgebras

3.1. **Universal compositions and cofree coalgebras.** Fix a symmetric monoidal category $(\mathcal{C},\otimes)$, and write Σ_* for the category whose objects are surjections $S \twoheadrightarrow *$ from a finite set to a singleton set and whose morphisms are compatible set isomorphisms (note: this is equivalent to just the category of finite sets and set isomorphisms, which is equivalent to the symmetric groups viewed as a category). Write $\Sigma_*\wr\Sigma_*$ for the category of whose objects are set surjections $S_1 \twoheadrightarrow S_2 \twoheadrightarrow *$ and whose morphisms are compatible set isomorphisms. More generally write the iterated wreath product $\underbrace{\Sigma_*\wr\cdots\wr\Sigma_*}_{n}$ for the category of surjections $S_1 \twoheadrightarrow \cdots \twoheadrightarrow S_n \twoheadrightarrow *$ and compatible set isomorphisms. There are functors ${\Sigma_*}^{\wr n} \longrightarrow {\Sigma_*}^{\wr m}$ given by composing surjections if $n>m$, and inserting isomorphisms if $m>n$. These fit together to give an augmented simplicial category with extra degeneracies.

$$\cdots \to \Sigma_*\wr\Sigma_*\wr\Sigma_*\wr\Sigma_* \to \Sigma_*\wr\Sigma_*\wr\Sigma_* \to \Sigma_*\wr\Sigma_* \to \Sigma_*$$

The dotted, leftwards arrows above are sections of their neighboring solid rightwards arrows. Write γ_n for the composition $\gamma_n : \Sigma_*\wr\cdots\wr\Sigma_* \to \Sigma_*$.

A symmetric sequence in $\mathcal{C}$ is a functor $A:\Sigma_*\to\mathcal{C}$. Given two symmetric sequences $A,B:\Sigma_*\to\mathcal{C}$, we define their universal composition product $A\circledcirc B : \Sigma_*\wr\Sigma_*\to\mathcal{C}$ by

$$(A\circledcirc B)(S_1 \xrightarrow{f} S_2 \xrightarrow{g} *) = A(S_2 \xrightarrow{g} *) \bigotimes \left(\bigotimes_{s\in S_2} B\big(f^{-1}(s) \xrightarrow{f} \{s\}\big)\right).$$

We define $A\circledcirc B\circledcirc C : \Sigma_*\wr\Sigma_*\wr\Sigma_* \to \mathcal{C}$ and etc. similarly. Note that $A\circledcirc B$ is not again a symmetric sequence. However we may construct a symmetric sequence from $A\circledcirc B$ by either left or right Kan extension over $\gamma_2 : \Sigma_*\wr\Sigma_*\to\Sigma_*$.

The left Kan extension $\mathrm{Lkan}_{\gamma_2} A\circledcirc B$ is the usual operadic composition product from the literature. In this language, an operad is a symmetric sequence equipped with a natural transformation $\mu:(A\circledcirc A)\to A\,\gamma_2$ such that the two induced natural transformations $\mu(\mu\wr\mathrm{Id}),\mu(\mathrm{Id}\wr\mu):(A\circledcirc A\circledcirc A)\rightrightarrows A\,\gamma_3$ are equal. Dually, a cooperad is a symmetric sequence with natural transformation $\Delta : A\,\gamma_2 \to (A\circledcirc A)$ so that the two induced natural transformations $\mathcal{A}\,\gamma_3 \rightrightarrows (A\circledcirc A\circledcirc A)$ are equal.

The universal composition product is associative, but its Kan extensions may not be. In most categories of interest (e.g. ungraded ring modules and topological spaces), associativity is lost on right Kan extension; though not on left Kan extension. Write $A\bullet B = \mathrm{Rkan}_{\gamma_2}(A\circledcirc B)$, $A\bullet B\bullet C = \mathrm{Rkan}_{\gamma_3}(A\circledcirc B\circledcirc C)$, etc.

In general $A \bullet (B \bullet C) \neq (A \bullet B) \bullet C$. However there are "parenthesization maps" from each of these to $A \bullet B \bullet C$. Much of the difficulty of coalgebras can be traced directly to this point.

Using this framework I have proven a new method for building cofree coalgebras extending and simplifying that of Justin Smith.

Theorem 3.1 (W—). *Let C be a cooperad and $V \in \mathrm{Ob}(\mathcal{C})$. The cofree C-coalgebra on V is given by the categorical limit of the diagram*

$$\begin{array}{ccccccc} & & C \bullet (C \bullet V) & & C \bullet \big(C \bullet (C \bullet V)\big) & & \cdots \\ & & \downarrow & & \downarrow & & \\ C \bullet V & \longrightarrow & C \bullet C \bullet V & \rightrightarrows & C \bullet C \bullet C \bullet V & \Rrightarrow & \cdots \end{array}$$

3.2. **Future work.** My description of cooperads and coalgebras via universal composition products has already proven useful in computations with the graph cooperad and its relationship with the Lie and associative cooperads. Conceptually, it has proven convenient to be able to work on the other side of Kan extension from the literature, and notationally it is much more clean to work with universal composition products rather than the cooperadic composition product. Further this work leads to a clear understanding of Michael Ching's construction of the topological cooperadic cobar construction.

In this vein, I plan to continue a reorganization of cooperad theory around universal composition products. In particular this leads to a new way of describing the cofree cooperad functor which can be used for a topological operadic bar construction. Applications would include constructions useful for my joint work with Sinha on Lie coalgebras, understanding the topological graph cooperad and its linear and Moore duals, as well as revisiting the work of Ching understanding Goodwillie's homotopy calculus of functors.

4. Calculus of graph functors

4.1. **Chromatic numbers via algebraic topology.** By a graph G we will mean an undirected graph with no multiple edges or loops. A graph map $G \to H$ is a map of vertex sets which preserves adjacency. A graph has an n-coloring if its vertices can be colored with n colors such that no adjacent vertices are colored the same. Note that an n-coloring of a graph G is equivalent to a graph map $G \to K_n$ to the complete graph on n vertices. Write $\chi(G)$ for the minimal number of colors required to color G.

In 1978 Lovász constructed homology obstructions to the existence of maps $G \to K_n$, and more recently his work was generalized by Babson, Kozlov, and others through the introduction of a bifunctor $\mathrm{Hom}(-,-)$ which assigns a topological space to a pair of graphs.

Definition 4.1. Let H be a graph. Define $\mathcal{P}(H)$, the power graph of H, to be the graph whose vertex set is $\mathcal{P}_0(V(H))$ with an edge between S and T if an only if there is an edge between every element of S and every element of T in H.

Given graphs G and H define the poset $\mathrm{hom(G,H)}$ to be the set of graph maps $G \to \mathcal{P}(H)$ with poset structure inherited from $\mathcal{P}(H)$.

Write $\mathrm{Hom(G,H)}$ for the realization of the poset $\mathrm{hom(G,H)}$.

$\mathrm{Hom}(-,-)$ is a bifunctor, contravariant in the first variable and covariant in the second. Babson and Kozlov build obstructions to graph maps using functoriality of $\mathrm{Hom}(-,-)$ as follows. A graph map $f : G \to K_n$ induces a map of spaces $\mathrm{Hom(K_2,f)} : \mathrm{Hom(K_2,G)} \to \mathrm{Hom(K_2,K_n)}$. Furthermore the antipodal map on K_2 gives a free $\mathbb{Z}_2$-action on the above spaces and makes $\mathrm{Hom(K_2,f)}$ a $\mathbb{Z}_2$-equivariant map. Considering $\mathrm{Hom(K_2,f)}$ as a map of $\mathbb{Z}_2$-bundles, the generalized Borsuk-Ulam theorem tells us that the image of the first nonvanishing Stiefel-Whitney class of $\mathrm{Hom(K_2,G)}/\mathbb{Z}_2$ must be nontrivial in $\mathrm{Hom(K_2,K_n)}/\mathbb{Z}_2$. However by a direct computation, $\mathrm{Hom(K_2,K_n)}$ is $(n-3)$-connected.

Theorem 4.2 (Lovász). *For any graph G, $\chi(G) \geq \mathrm{connectivity}\big(\mathrm{Hom(K_2,G)}\big) + 3$.*

There is nothing particularly important about the "test graph" K_2 used above aside from it having a free group action. Babson and Kozlov replaced it by the cyclic graph C_{2k+1} with the following result.

Theorem 4.3 (Babson and Kozlov)**.** *For any graph G,* $\chi(G) \geq \mathrm{connectivity}\big(\mathrm{Hom}(C_{2k+1}, G)\big) + 4$.

In practice, the above constructions seem to give fairly sharp lower bound for chromatic numbers. The main obstacle to quick chromatic number calculations appears to be that it is very computationally intensive to extract connectivity information for general graphs. The work required to compute $\mathrm{Hom}(C_{2k+1}, K_n)$ above is a triumph of fearless combinatorial mathematics.

4.2. **A calculus of graph functors.** Brian Munson and I have begun the construction of a calculus of graph functors approximating covariant functors $F : \mathcal{G}raphs \to \mathcal{T}op$ which are continuous in the sense that the evaluation map $\mathrm{Hom}(G, H) \times F(G) \to F(H)$ is a continuous map. The functor $\mathrm{Hom}(K, -)$ is one such. The construction of our calculus is inspired by Weiss' orthogonal calculus of functors from vector spaces to topological spaces which are continuous in the sense that $\mathrm{mor}(V, W) \times F(V) \to F(W)$ is continuous. Our work relies on the fact that $\mathrm{hom}(-, -)$ is essentially an enrichment of the category of graphs over the category of posets.

The end result of our work is, for every graph functor F, an approximating tower of fibrations of graph functors $\cdots \to P_3F \to P_2F \to P_1F$ where the fibers all have a particular simple form, and going up the tower better approximates good functors F in the sense that a natural transformation $F \to P_nF$ becomes highly connected. In this case, we should be able to read off connectivity information for $\mathrm{Hom}(K, G)$ from the connectivity of the fibers in the bottom of the approximating tower of $P_n\mathrm{Hom}(K, -)$ evaluated at G.

Working analogous to Weiss, we have completed roughly half of the constructions required to reach this goal. At present, we have worked out the universal property which should define the n-th polynomial approximation P_nF (which involves a poset-enriched homotopy colimit), we can show that polynomial of degree n implies polynomial of degree $n-1$, and we can show that approximating towers for functors exist. We have also completed the initial constructions towards understanding fibers of approximating towers.

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